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ABSTRACT

This paper briefly explains the logic underlying the basic calculations employed in canonical correlation analysis. A small hypothetical data set is employed to illustrate that canonical correlation analysis subsumes both univariate and multivariate parametric methods. Several real data sets are employed to illustrate other themes. Three common fallacious interpretation practices that may lead to incorrect conclusions are discussed; these fallacies affect the interpretation of function coefficients, interpretation of redundancy coefficients, and failure to partition using canonical commonality analysis. The use of the factor analytic method of rotation to simplify results is also discussed. It is suggested that canonical correlation analysis is a powerful analytic method that frequently best honors the complex nature of the reality about which the researcher wishes to generalize. Thirty-two tables and two graphs are presented. (Author/TJH)

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Canonical Correlation Analysis:

An Explanation with Comments on Correct Practice

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ABSTRACT

The paper briefly explains the logic underlying the basic calculations employed in canonical correlation analysis. A small hypothetical data set is employed to illustrate that canonical correlation analysis subsumes both univariate and multivariate parametric methods. Several real data sets are employed to illustrate other themes. The paper discusses three common fallacious interpretation practices that may lead to incorrect conclusions based on canonical results. The use of rotation to simplify results is discussed. It is suggested that canonical correlation analysis is a powerful analytic method that frequently best honors the complex nature of the reality about which the researcher wishes to generalize.

Several trends in analytic practice are discernable as incremental changes that are moving social science slowly toward more productive inquiry. For example, researchers have increasingly recognized that statistical significance may not be a particularly effective criterion with which to evaluate results (Thompson, 1987c; 1988); popular developments in meta-analysis (Jones & Fiske, 1953; Glass, McGaw & Smith, 1981; Rosenthal, 1984) may have compelled more researchers to recognize the importance of effect sizes in their studies. Researchers have also increasingly recognized that statistical control, such as that employed in analysis of covariance (ANCOVA), must be used with extraordinary caution; these methods tend to either be unnecessary or seriously distort results (Thompson, in press-c) and can lead to "tragically misleading analyses" (Campbell & Erlebacher, 1975, p. 597).

However, the trend away from the use of classical analysis of variance methods (Goodwin & Goodwin, 1985) may be the most noteworthy trend of all, since the use of the methods can have several deleterious effects (Cohen, 1968; Thompson, 1986a). Even when analysis of variance methods represent good analytic choices, regression or general linear model approaches to the methods using a priori contrast coding still tend to be superior since these approaches tend to yield greater power against Type II error and tend to be more theoretically grounded (Thompson, 1985a; 1987b).

The gradual shift away from the use of analysis of variance approaches has been due in part to an increased recognition that all parametric univariate methods are special cases of regression analysis (Cohen, 1968). The shift has also been due to increased recognition that many researchers

> prefer experimental over correlational research designs because experimental designs provide more complete information about causality. Why does this situation contribute to OVAism? Because some researchers confuse research designs with the statistical techniques which are used to analyze the data which the designs help to generate. (Thompson, 1981, p. 5)

As Thompson (in press-c) notes,

> The fact that OVA methods are appropriate when predictor variables such as experimental assignment naturally occur at the nominal level of scale has stimulated some researchers to unconsciously [and incorrectly] associate the consequences of experimental design selection with OVA methods.

However, in reality all parametric significance tests, including those which are multivariate, are special cases of canonical correlation analysis (Knapp, 1978). Indeed, Thompson (1985b) illustrates how various univariate and multivariate analyses can all be conducted using canonical correlation analysis. Thompson (1986b) notes that the evaluation of several hypothesis tests within a single study inflates the experimentwise Type I error probability, usually to a somewhat unknown degree. The failure to use multivariate methods often also distorts the reality about which the researcher is attempting to generalize--the least of these distortions occurs when a researcher completes several univariate tests and finds no statistically significant results when significance would have occurred if a multivariate test had been employed. Thompson (1986b) presents a data set illustrating how this can occur. These various considerations suggest that canonical correlation analysis may be a powerful and important weapon in the social scientist's arsenal of analytic weapons.

The purpose of the present paper is to briefly explain the logic underlying the basic calculations employed in canonical correlation analysis. The paper also employs a small hypothetical data set to demonstrate that canonical correlation analysis subsumes both univariate and multivariate parametric methods. Three common fallacious interpretation practices that may lead to incorrect conclusions based on canonical results are discussed. The use of rotation in the canonical case is illustrated and briefly discussed.

The Basic Logic of Canonical Calculations

Thompson (1983) notes that canonical correlation can be presented in bivariate terms. This conceptualization has instructional appeal because most students feel comfortable working with bivariate correlation coefficients. The view is also important because it forces realization that canonical analysis, like all parametric methods, involves the creation of "synthetic" scores for each person. In regression analyses the synthetic scores are the predicted dependent variable scores of each of the subjects, sometimes termed "YHAT"; the correlation between the subjects' actual dependent variable scores and synthetic dependent variable ("YHAT") scores is the multiple correlation coefficient, while the sum of squares of the "YHAT" scores equals the sum of squares explained. In factor analysis these synthetic variables are the factor scores of each subject on each of the factors. In discriminant analysis these synthetic variables are the discriminant scores of each subject on each of the discriminant functions.

Table 1 presents a hypothetical data set (Thompson, 1987a) that will be employed to illustrate how scores of individuals are converted into the synthetic variables that are actually the focus of a canonical correlation analysis. The data are adapted from those presented by Harris (1987). The data set involves two criterion variables, "X" and "Y," and two predictor variables, "A" and "B". Since canonical correlation analysis presumes at least two predictor and at least two criterion variables, the data set represents the simplest case for which a true canonical analysis can be conducted. If a canonical analysis of a smaller data set was conducted, most researchers would refer to the analysis using some other name, such as multiple regression analysis. Table 1 also presents each of the five persons' scores on the four variables converted into their equivalent _Z_-score forms. Table 2 presents the SPSS-X program used to analyze these data; the reader may wish to replicate this analysis to reflect in more detail on the results reported here.

INSERT TABLES 1 AND 2 ABOUT HERE.

Various analytic methods yield weights that are applied to variables to optimize some condition--such weights include beta weights, factor pattern coefficients, and discriminant function coefficients. These weights are all equivalent, but in canonical correlation analysis the weights are usually labelled standardized function coefficients. These weights are applied to each individual's data to yield the synthetic variables that are the basis for canonical analysis.

However, in canonical analysis several sets of weights and of the resulting synthetic variables can be created. These canonical functions are related to factors, are uncorrelated or orthogonal, and can be rotated in various ways (Thompson, 1984). The number of functions that can be computed in a canonical analysis equals the number of variables in the smaller of the two variable sets, as explained by Thompson (1984). In the present example, since each variable set consisted of two variables, two canonical functions could be computed. Table 3 presents the canonical function coefficients and other selected results from the analysis.

<u>INSERT TABLE 3 ABOUT HERE.</u>

Table 4 illustrates the computation of the synthetic variables for each of the five subjects using the Function I function coefficients; the reader may wish to compute the corresponding values associated with the Function II results. For a given function, two synthetic scores are produced for each subject--one associated with the composite of weighted criterion variables, and one associated with the composite of weighted predictor variables. For example, as noted in Table 4, the criterion synthetic variable score, "CRITCOMP," for subject one was 1.29589 ([-1.44986*-1.35287] + [+1.04101*-.63850]). By the same token, the predictor synthetic variable score for subject five was -1.21913 ([-1.58021*+1.32563] + [1.24215*+.67606]).

INSERT TABLE 4 ABOUT HERE

The canonical correlation (Rc) is nothing more (or less) than the Pearson product-moment correlation between the synthetic variable scores of the subjects on a given function. This can be illustrated in several ways using the present results. For example, for this case, the bivariate correlation equals the sum of the cross-products of the two variables, the sum then being divided by $n - 1$. The cross products of the synthetic variables for each of the five subjects are presented in Table 4, as is the sum of these cross products. The sum divided by $n - 1$ (3.999947/4) equals, within rounding error, the actual Rc result reported in Table 3 for Function I.

An alternative presentation is graphic. Figure 1 presents the scattergram in which the five pairs of synthetic variable scores from Table 4 are arrayed. For example, note that the first subject's composite scores in Table 4 indicate that this subject is represented by the asterisk in the upper right position within the scattergram. Figure 1 also presents the least squares regression line best fitting these asterisks. In the two variable case, since the synthetic variables have means of zero, the slope of this regression line equals a beta weight, also equals the bivariate correlation between the synthetic variables, also equals the canonical correlation coefficient, i.e., .99999.

INSERT FIGURE 1 ABOUT HERE.

Table 5 presencs computations that illustrate the meaning of two other canonical results, structure coefficients and index coefficients. Structure coefficients have the same meaning in a canonical analysis as in other analyses, i.e., structure coefficients are bivariate correlation coefficients between a given criterion or predictor variable and the synthetic variable involving the variable set to which the variable belongs. For example, since "ZX" was a criterion variable, the correlation between "ZX" and "CRITCOMP" is the structure coefficient for "ZX." Note that the sum of the cross products of "ZX" and "CRITCOMP", labelled "XSTRUC" in Table 5, once divided by n - 1, equals within rounding error the structure coefficient for "ZX" presented in Table 3. An index coefficient is the correlation coefficient between a variable and the synthetic variable consisting of variables from the variable set to which the variable does not belong. Table 5 illustrates the calculation of the index coefficient for "ZX" on Function I. Thompson (1984) discusses the importance of index coefficients in greater detail.

INSERT TABLE 5 ABOUT HERE.

Canonical Correlation Analysis (CCA) as a General Method

In a seminal article, Cohen (1968, p. 426) noted that ANOVA and ANCOVA are special cases of multiple regression analysis, and argued that in this realization "lie possibilities for more relevant and therefore more powerful exploitation of research data." Since that time researchers have increasingly recognized that conventional multiple regression analysis of data as they were initially collected (no conversion of intervally scaled independent variables into dichotomies or trichotomies) does not discard information or distort reality; conventional regression analysis can be particularly useful when multiplicative effects are evaluated (e.g., through the use of powered vectors or product terms (but see Pedhazur, 1982, pp. 427-430)) or when commonality analyses are conducted (e.g., Thompson, 1985a)). Discarding variance is not generally good research practice (Thompson, in press-b). As Kerlinger (1986, p. 558) explains,

> ...partitioning a continuous variable into a dichotomy or trichotomy throws information away... To reduce a set of values with a relatively wide range to a dichotomy is to reduce its variance and thus its possible correlation with other variables. A good rule of research data analysis, therefore, is: <u>Do not reduce continuous variables to partitioned variables</u> (dichotomies, trichotomies, etc.) unless compelled to do so by circumstances or the nature of the data (seriously skewed, bimodal, etc.).

OVA methods (ANOVA, ANCOVA, MANOVA and MANCOVA) do not discard variance only when independent variables are already nominally scaled. Even in these cases, however, the regression implememt .ion of OVA methods using the <u>a priori</u> contrast coding explained by researchers such as Pedhazur (1982, chapters 9-14) and Loftus and Loftus (1982, chapter 15) has two important benefits, as explained in some detail by Thompson (1987b). First, <u>a priori</u> methods have more power against Type II error than do post hoc tests (e.g., Kirk, 1968, p. 96--Thompson (1987b, pp. 10-

11) catalogs similar statements). The exception is when all ways or factors have only two levels--then, and only then, both *a priori* and post hoc tests are superfluous since each statistically significant omnibus hypothesis can only have occurred by a given pair of means being different.

Second, the use planned or *a priori* comparisons tends to force the researcher to be more thoughtful in conducting research. As Snodgrass, Levy-Berger and Haydon (1985, p. 386) suggest, "the experimenter who carries out post hoc comparisons often has a rather diffuse hypothesis about what the effects of the manipulation should be." Similarly, Keppel (1982, p. 165) notes that,

> planned comparisons are usually the motivating force behind an experiment. These comparisons are targeted from the start of the investigation and represent an interest in particular combinations of conditions--not in the overall experiment.

Indeed, *a priori* tests are often employed in lieu of omnibus tests in both univariate OVA (Hays, 1981, p. 426; Kirk, 1968, p. 73) and multivariate OVA (Swaminathan, in press) applications.

These various realizations have led to less frequent use of OVA methods (Goodwin & Goodwin, 1985), and to more frequent use of *a priori* contrast coding and regression approaches when OVA analyses are still conducted (Willson, 1982). However, canonical correlation analysis, and not regression analysis, is the most general case of the general linear model (Baggalley, 1981, p. 129). Fornell (1978, p. 168) notes that "multiple regression, MANOVA and ANOVA, and multiple disriminant analysis can all be shown to be special cases of canonical analysis. Principal components analysis is also in the inner family circle." In an important article, Knapp (1978, p. 410) demonstrated this in some mathematical detail and concluded that "virtually all of the commonly encountered tests of significance can be treated as special cases of canonical correlation analysis."

Thompson (1985b) employed the data presented in Table 6 to illustrate these identities. Various combinations of variables were analyzed using both canonical correlation analysis and more commonly used names for parametric methods (e.g., *t*-test, ANOVA, MANOVA) to show that canonical analysis can be used to yield results from both conventional univariate and multivariate methods. The results in the Thompson (1985b) report were generated using SPSS version 9.2.

INSERT TABLE 6 ABOUT HERE.

In the present paper similar analyses were conducted using the SAS file presented in Table 7. SAS allows the researcher to force the computer to analyze more results using multivariate approaches, while SPSS-X now arbitrarily defaults to univariate approaches to univariate data analyses. Thus, the equivalent results produced by the SAS package allows comparisons of results across methods with fewer steps in the comparison process. The reader may wish to replicate these analyses in order to make a more detailed comparison. Throughout the present paper results are presented to the same number of decimal places yielded by the SAS analysis.

INSERT TABLE 7 ABOUT HERE.

Table 8 presents an analysis illustrating the equivalence of _t_-tests and canonical correlation analysis. The _p_ calculated value associated with the test c 'lifferences in means on variable _Y_ across variable _B_ groups 'J' and "1" was 0.2149. Table 8 also presents results from a canonical correlation analysis involving variable _Y_ related with variable _B_, which in this case was also a dummy coding column. The resulting _p_ calculated value was 0.2149.

INSERT TABLE 8 ABOUT HERE.

Table 9 presents a conventional product-moment analysis of the bivariate relationship between variables _Y_ and _A_. The correlation coefficient was computed to be 0.56643 with an associated _p_ calculated value of 0.0548. A canonical correlation analysis yielded a _Rc_ value of 0.566434 with an associated _p_ calculated value of 0.0548.

INSERT TABLE 9 ABOUT HERE.

Table 10 presents a conventional 2x3 factorial ANOVA involving scores on the dependent variable _Y_ across ways defined by variables _APRIME_ and _B_. Table 11 presents results from four separate canonical correlation analyses using different combinations of the _a priori_ contrast coding expressions of the information involved in the variables _APRIME_ and _B_. It is noteworthy that the correlation ratio computed for the error effect for the full ANOVA model presented in Table 10 was 0.426573 (61.0/143.0); the lambda value presented in Table 11 associated with all contrasts was 0.42657343. The result is not surprising since multivariate lambda is analogous to the univariate sum-of-squares error divided by the SOS total.

INSERT TABLES 10 AND 11 ABOUT HERE.

Table 12 converts the canonical lambda's into seperate effects for each ANOVA omnibus effect. Smaller lambda's connote larger effect sizes. The APRIME main effect reported in Table 10 has the largest effect size 0.391608 (56.0/143.0), thus the smallest lambda reported in Table 12 (0.52136752) is associated with the same main effect. Table 13 converts the Table 12 omnibus effect lambda's into ANOVA *F* tests comparable to those presented in Table 10.

INSERT TABLES 12 AND 13 ABOUT HERE.

Table 14 presents the multiple regression analysis in which variables, *X*, *A*, and *B* are used to predict dependent variable *Y*. Table 14 also presents results from the canonical correlation analysis involving the same two variable sets. The two sets of results are directly comparable; the only difference is that the canonical analysis yields the equivalent results presented to more digits to the right of the decimal. Table 15 illustrates the conversion of beta weights into canonical function coefficients and vice versa. Thompson and Borrello (1985) discuss these relationships in more detail.

INSERT TABLES 14 AND 15 ABOUT HERE.

Table 16 presents results from a discriminant analysis involving use of variables *Y* and *X* to predict membership in the three groups delineated by the variable APRIME. The table also presents results from the canonical correlation analysis involving the variables *Y* and *X* and the dummy coding variables, BT1 and BT2, which express in a different form exactly the same information contained in APRIME. The results are directly comparable.

INSERT TABLE 16 ABOUT HERE.

Table 17 presents the function coefficients for variables Y and X produced by both analyses for both functions I and II. In order to compare these results, the largest coefficient in each function is set equal to unity. Tatsuoka (1971, pp. 177-183) explains this conversion and notes that he first discussed the equivalence of these methods 35 years ago (Tatsuoka, 1953). The identities illustrated here and summarized by Knapp (1978) have been known for some time, but the implications of these identities have not always been appreciated by researchers.

INSERT TABLE 17 ABOUT HERE.

Table 18 presents the results of a 2x3 factorial MANOVA involving dependent variables Y and X and the classification variables APRIME and B. Table 19 presents results from four separate canonical correlation analyses involving the classification variables expressed as the orthogonal constrasts A1, A2, B1, A1B1, A2B1. Table 20 presents the conversion of the Table 19 results to lambda values associated with the omnibus MANOVA effects presented in Table 18. The Table 18 and 20 lambda's are comparable. Zinkgraf (1983) provides additional examples of these relationships.

INSERT TABLES 18, 19, AND 20 ABOUT HERE.

These comparisons should not be taken to mean that special cases of canonical methods will always yield the same results as the more general canonical methods. For example, the results will be different if a researcher performs a canonical analysis with unchanged variables as against converting some variables to the nominal level of scale in order to do an OVA analysis. Canonical correlation omnibus, simultaneous analysis of a multivariate data set may yield very different conclusions from several univariate analyses of the same data set, even as regards whether results are found to be statistically significant (e.g., Thompson, 1986b). Finally, when ways of a design have more than two levels, use of general linear model methods and a priori contrasts can yield different conclusions than those produced by special cases of canonical analysis called by names such as MANOVA. In each of these cases the use of canonical correlation analysis would be preferable.

The comparisons do illustrate that canonical correlation analysis subsumes other parametric methods as special cases. This realization has heuristic value. Canonical correlation analysis provides a framework within which other parametric methods can be related. The realization that OVA and other methods are special cases of canonical correlation analysis should give researchers pause to think that the more general methods should be employed more often to avoid the discarding of variance that many researchers perform in order to conduct OVA analyses. In reality, all studies are correlational in the sense that even studies with both experimental design and OVA analyses are about "the job of stating and testing more or less general relationships between properties of nature" (Homans, 1967, p. 7). In experimental studies degrees of relationship are expressed as effect size estimates such as the correlation ratio or omega-squared.

Three Common Interpretation Fallacies

Canonical correlation analysis is a potent analytic method. But the difficulty of interpreting canonical results can challenge even the most seasoned analyst. As Thompson (1980, pp. 1, 16-17) notes, one

> reason why the technique is rarely used involves the difficulties which can be encountered in trying to interpret canonical results... The neophyte student of canonical correlation analysis may be overwhelmed by the myriad coefficients which the procedure produces... [But] canonical correlation analysis produces results which can be theoretically rich, and if properly implemented the procedure can adequately capture some of the complex dynamics involved in educational reality.

However, the interpretation of canonical results can be facilitated if three common interpretation fallacies are avoided.

Interpretation of Function Coefficients

In an artificial world of forced-choices, the analyst might interpret structure coefficients while ignoring function coefficients. Structure coefficients are the most helpful coefficients to consult when interpreting canonical results, although many researchers do not interpret and some do not even report structure coefficients. Since structure coefficients inform the researcher of the correlation between each variable and the synthetic variables, these coefficients are what inform the researcher regarding the meaning of what is actually being correlated in a given analysis.

As noted previously, structure coefficients have the same meaning in the canonical cases as in the other analytic methods that the canonical methods subsume as special cases. For example, in principal components analysis the correlation between the scores on one variable and the factor scores on one factor is the structure coefficient for that variable on that factor. And as Gorsuch (1983, p. 207) notes, "the basic matrix for interpreting the factors is the factor structure." Similarly, in a discriminant analysis, the correlation between the scores on a predictor variable and the discriminant function scores on a given function is the structure coefficient for that variable on that function.

In the regression case, the correlation between scores on a predictor variable and the "YHAT" scores is the structure coefficient for the predictor variable. Just as structure coefficients are vitally important in interpreting results in other analytic cases, structure coefficients can be very important in interpreting multiple regression results (Cooley & Lohnes, 1971, pp. 54-55). Thompson and Borrello (1985) present an explanation of this application and an actual research example in which the interpretation solely of beta weights rather than of structure coefficients would conceivably have lead to incorrect conclusions.

Thus, with respect to canonical analysis, Meredith (1964, p. 55) suggested that, "If the variables within each set are moderately intercorrelated the possibility of interpreting the canonical variates by inspection of the appropriate regression weights [function coefficients] is practically nil." Similarly, Kerlinger and Pedhazur (1973, p. 344) argued that, "A canonical correlation analysis also yields weights, which, theoretically at least, are interpreted as regression [beta] weights. These weights [function coefficients] appear to be the weak link in the canonical correlation analysis chain." Levine (1977, p. 20, his emphasis) is even more emphatic:

> I specifically say that one <u>has</u> to do this [interpret structure coefficients] since I firmly believe as long as one wants information about the nature of the canonical correlation relationship, not merely the computation of the [synthetic function] scores, one must have the structure matrix.

The hypothetical results presented in Table 3 illustrate that the interpretation of only function coefficients can lead to seriously distorted conclusions. The standardized function coefficients might lead the naive analyst to conclude that all four variables contribute appreciable information to the relationship between the two sets of synthetic variable scores on Function I. In reality, variables "ZY" and "ZB" share almost no variance at all with the function's scores.

The realization that multiple regression analysis is a special case of canonical correlation analysis suggests that structure coefficients may also be important aids to interpretation in the regression case, as Thompson and Borrello (1985) argued. The data reported here can also be employed to illustrate this point. Assume that the synthetic variable CRITCOMP was not a synthetic variables, but a Z-score expression of an actual measure. Table 21 presents the regression results associated with the prediction of CRITCOMP with variables A and B. The tabled beta weight (1.242139) should not be taken to indicate that variable B shares appreciable variance with the "YHAT" scores in this case. In fact, B only shares 0.0324% (0.018 x 0.018) of its variance with "YHAT", though B is useful in creating the "YHAT" scores.

INSERT TABLE 21 ABOUT HERE.

These data involve the presence of a variable, B, that "suppresses" the relationship between A and CRITCOMP. This can be illustrated for these data by computing the correlation coefficient (-0.999822) between CRITCOMP and A residualized for the influences of B (ZARESI in the Table 2 command file). The comparisons of function and structure coefficients for variables alerts the researcher to the existence of such dynamics. In an artificial forced-choice world in which only one coefficient could be consulted, structure coefficients might be preeminent; in the real world both coefficients should be consulted in interpretation. Interpretations based solely on function coefficients should be eschewed.

Interpretation of Redundancy Coefficients

If the squared structure coefficients for a given set of variables are added and then the sum is divided by the number of variables in the set, the result informs the researcher regarding how much of the variance in the variables, on the average, is contained within the synthetic scores for that function. This result is called a variate adequacy coefficient (Thompson, 1984). Stewart and Love (1968) suggested that multiplying the adequacy coefficient times the squared canonical correlation yields a coefficient that they labelled a redundancy coefficient (Rd). Miller (1975) developed a partial test distribution to test the statistical significance of redundancy coefficients. Cooley and Lohnes (1976, p. 212) suggest that redundancy coefficients have great utility. In reality, the interpretation of redundancy coefficients does not make much sense in a conventional canonical analysis.

As Cramer and Nicewander (1979) proved in detail, redundancy coefficients are not truly multivariate. This is very disturbing, because the main argument in favor of multivariate methods (for both substantive and statistical reasons) is that these methods simultaneously consider all relationships during the analysis (Thompson, 1986b).

Table 22 helps to illustrate the problem. The table presents the adequacy, redundancy, and squared Rc's for both functions for the hypothetical Table 1 data, as well as the pooled values. For example, the pooled redundancy coefficient for the criterion variable set is 0.242783. Table 23 presents the results of four regression analyses for various criterion variables and predictor variable sets. The table illustrates that the average squared multiple R for a variable set equals the pooled redundancy coefficient for that variable set. The redundancy coefficient is the average of a set of univariate results!

INSERT TABLES 22 AND 23 ABOUT HERE.

A redundancy coefficient for a given variable set on a given function equals the adequacy coefficient for the set times the squared Rc for the function. The redundancy coefficient can only equal one when the synthetic variables for the function represent all the variance of every variable in the set, and the squared Rc also exactly equals one. This does not usually occur in practice. Thus, redundancy coefficients are useful only to test outcomes that rarely occur and which may be unexpected (Thompson, 1980, p. 16; Thompson, 1984). Furthermore, it seems contradictory to employ an analysis that use functions coefficients to optimize Rc, and then to interpret results not optimized as part of the analysis, i.e., redundancy coefficients.

However, there are exceptions to most rules. Table 24 presents the correlation matrix associated with a concurrent validity study conducted by Sexton, McLean, Boyd, Thompson and McCormick (in press). Table 25 presents the results of a canonical correlation analysis of the Table 24 data. In this case variate adequate coefficients for both variates on function I were quite large, and the squared Rc was also remarkably large. Thus, in this rather unusual case, the Rd coefficients presented in Table 25 were impressively large on function I. It may be more reasonable in concurrent validity studies to expect such results, but, again, such results are not usually expected.

INSERT TABLES 24 AND 25 ABOUT HERE.

Failure to Partition Using Canonical Commonality Analysis

Researchers have been aware for some time that interpretation of regression results is often facilitated by conducting "commonality analyses" (Newton & Spurrell, 1967; Thompson, 1985a). These analyses partition variance to indicate how much variance is unique to a given variable, and how much variance is common to other variables. As an example analysis for the regression case, Seibold and McPhee (1979, pp. 364-365) present a cancer study in which the results would have been grossly misinterpreted if a commonality analysis had not been conducted.

Given that multiple regression is a special case of canonical correlation analysis, it seems reasonable to expect that the same variance partitioning procedures might also be useful in the true canonical case. Thompson and Miller (1985) explain the multivariate procedure using an actual research example in which educators' perceptions of dying students and of death were investigated. The procedure may be very useful in research situations in which at least one of the variable sets consists of variables that are conceptually or theoretically distinct. As in the regression case, <u>the failure to employ commonality analysis can lead to less informed interpretation of results</u>.

The Table 24 data can be employed to illustrate the mechanics of the procedure. Let us assume (rather artificially) that the Battelle Developmental Inventory consisted of three conceptually or empirically distinct sets of scales: (a) the Social (So) scale; (b) the Adaptive (Ad) and the Motor (Mo) scales; and (c) the Communication (Co) and the Cognitive (Cg) scales. Table 26 presents the squared $\underline{R}c$'s associated with use of different combinations of the three variable sets to predict criterion variate composite scores on function I. These values are generated by using COMPUTE statements to produce canonical variate scores (see Table 2 for an illustration) and then using regression procedures to predict the synthetic composite scores with different combinations of variables. Table 27 illustrates the calculation of multivariate commonality coefficients for these data, given the results presented in Table 26.

INSERT TABLES 26 AND 27 ABOUT HERE.

Table 28 presents the coefficients in the format typically employed in reports of commonality analyses. In the present case, the Table 28 results indicate that almost all of the predictive power of the three artificial sets of predictor variables is common to all three sets. This result is not surprising, given redundancy analysis suggesting the the variable sets are characterized by "g" variates creating a "g" function.

INSERT TABLE 28 ABOUT HERE.

Multivariate commonality analyses can be useful when a variable set consists of theoretically or empirically distinct sets of variables. Several authors present the procedures for computing commonality coefficients for different numbers of variable sets (Cooley & Lohnes, 1976, p. 222; Seibold & McPhee, 1979, p. 358); these procedures can be generalized to the multivariate case in the manner already illustrated. Once results like those presented in Table 26 are available, microcomputer "spreadsheet" software is useful for performing the remaining computations, but mainframe statistical packages can also be employed. Table 29 presents an SPSS-X file like that used to generate the Thompson and Miller (1985) analysis. The reader may wish to replicate these computations using the program.

INSERT TABLE 29 ABOUT HERE.

A Note About Rotation

Space precludes complete discussion of the many variations on canonical methods, including methods that can be employed with more than two variable sets (Horst, 1961), methods that do optimize or at least consider redundancy (DeSarbo, 1981; Johansson, 1981; Wollenberg, 1977), and methods for eliminating variables by consulting communality (Thompson, 1984, pp. 47-51) or other values so that results will be more parsimonious and generalizable (Rim, 1972). Space also precludes detailed discussion of ways to estimate the sample size required for reasonable power in a canonical correlation analysis, but Figure 2 is presented to illustrate some possibilities for evaluating power in the canonical case. The figure is part of a printout generated by software described by Thompson (in press-a).

INSERT FIGURE 2 ABOUT HERE.

However, the linkage between canonical correlation analysis and factor analysis suggests that rotation, which is so useful in factor analysis, may be useful in the canonical case as well. Some discussion of these applications is warranted. Thompson (1984, pp. 31-41) provides additional discussion of various rotation considerations.

Table 30 presents selected canonical results associated with the study reported by Webber, Thompson and Berenson (1987/1988).

This analysis represents the special case in which canonical function and structure matrices exactly equal each other, since orthogonal factor scores were the basis for the analysis. Whenever all variables in a set are perfectly uncorrelated, then function and structure matrices are equivalent.

INSERT TABLE 30 ABOUT HERE.

Rotation to the varimax or similar criteria is typically not appropriate in the canonical case, because these methods ignore the fact that canonical analysis involves two distinct variable sets. As Thorndike (1976, p. 4) argues,

> The two sets of variables presumably have been kept separate for a reason. If an investigator is interested in the structure of the combined sets, then he probably should have performed a traditional factor analysis in the first place.

However, Bentler and Huba (1982) propose a rotation strategy that honors membership in variable sets. Huba, Palisoc and Bentler (1982) present a computer program that implements the method. Table 31 presents a simultaneous orthogonal rotation of the Table 30 results.

INSERT TABLE 31 ABOUT HERE.

Table 32 presents the correlation coefficients among variate scores. The post-rotation canonical correlation coefficients are presented on the diagonal of the matrix. A comparison with the coefficients presented in Table 30 indicates that the rotation distributed some of the variance from the first two functions onto the third function as a way of achieving a simpler structure.

INSERT TABLE 32 ABOUT HERE.

The software provided by Huba, Palisoc and Bentler (1982) also computes an orthogonally rotated maximum likelihood factor analysis using canonical results. Webber, Thompson and Berenson (1987/1988) present these results for these data.

It is not yet clear whether rotated canonical results produce interpretations that are more generalizable or less sample specific. Theoretically, it might be argued that results with simpler structure are more parsimonious and therefore should be more generalizable. Monte Carlo work is needed to explore this issue. However, it should be noted that canonical analyses are conducted to optimally weight variables from two variable sets so that synthetic scores composed from the sets will be maximally related on the first function, and next most maximally related on each subsequent and orthogonal function. Although the sum of the squared canonical correlations remains the same both before and after rotation (see Thompson, 1984, pp. 33-38), rotation of canonical results inherently violates, to some degree, the basic logic of the methods, because variance is distributed across individual functions.

Summary

The logic underlying the basic calculations employed in canonical correlation analysis has been explained. Three common fallacious interpretation practices that may lead to incorrect conclusions based on canonical results were presented. A small hypothetical data set was employed to make the discussion concrete.

Notwithstanding some opinion to the contrary (Kerlinger, 1986, p. 606; Thompson, 1987d), canonical correlation analysis is a powerful analytic method that frequently best honors the complex nature of the reality about which the researcher wishes to generalize. As Kerlinger (1973, p. 652) suggests, "some research problems almost demand canonical analysis." Similarly, Cooley and Lohnes (1971, p. 176) suggest that "it is the simplest model that can begin to do justice to this difficult problem of scientific generalization."

More researchers need to recognize the value of multivariate methods in general and of canonical correlation analysis in particular. Tatsuoka's (1973, p. 273) previous remarks remain telling:

> The often-hearn argument, "I'm more interested in seeing how each variable, in its own right, affects the outcome" overlooks the fact that any variable taken in isolation may affect the criterion differently from the way it will act in the company of other variables. It also overlooks the fact that multivariate analysis--precisely by considering all the variables simultaneously--can throw light on how each one contributes to the relation.

However, the potentials of canonical correlation analysis will only be realized if researchers understand the logic underlying the method and if some serious interpretation pitfalls are avoided.

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Table 1
Hypothetical "Bird Beak" Data

X	Y	A	B	ZX	ZY	ZA	ZB
9.0	7.0	10.0	8.0	-1.35287	-.63850	.00000	1.04326
11.0	6.0	10.0	5.0	-.44490	-1.26448	.00000	-.53744
12.0	8.0	8.0	4.0	.00908	-.01252	-.81650	-1.06434
13.0	10.0	8.0	5.0	.46306	1.23944	-.81650	-.53744
14.9	9.1	14.0	8.1	1.32563	.67606	1.63299	1.09595

Table 2
SPSS-X Command File Used to Analyze the Table 1 Data

```
TITLE 'ANALYSIS OF RICHARD HARRIS DATA *****'
FILE HANDLE RJH/NAME='SWSMEP.DAT'
DATA LIST FILE=RJH/X 1-3 (1) Y 5-7 (1) A 9-11 (1) B 13-15 (1)
COMPUTE ZX=(X-11.98)/2.20273
COMPUTE ZY=(Y-08.02)/1.59750
COMPUTE ZA=(A-10.00)/2.44949
COMPUTE ZB=(B-06.02)/1.89789
COMPUTE REGYHAT=(-1.580199*ZA)+(1.242139*ZB)
COMPUTE ZARESI=ZA-(.774382*ZB)
COMPUTE CRITCOMP=(-1.44986*ZX)+(1.04101*ZY)
COMPUTE PREDCOMP=(-1.58021*ZA)+(1.24215*ZB)
COMPUTE XSTRUC=ZX*CRITCOMP
COMPUTE XINDEX=ZX*PREDCOMP
PRINT FORMATS ZX TO XINDEX(F8.5)
LIST VARIABLES=ALL/CASES=50
CONDESCRIPTIVE X TO XINDEX
STATISTICS ALL
SUBTITLE 'SHOW r's AMONG SYNTHETIC VARIABLES ARE OTHER COEFS'
PEARSON CORR X TO XINDEX
SUBTITLE 'COMPUTE R's TO SHOW Rd's ARE NOT MULTIVARIATE'
REGRESSION VARIABLES=X TO B/CRITERIA=TOLERANCE(.0001)/
  DEPENDENT=X/ENTER A B
REGRESSION VARIABLES=X TO B/CRITERIA=TOLERANCE(.0001)/
  DEPENDENT=Y/ENTER A B
REGRESSION VARIABLES=X TO B/CRITERIA=TOLERANCE(.0001)/
  DEPENDENT=A/ENTER X Y
REGRESSION VARIABLES=X TO B/CRITERIA=TOLERANCE(.0001)/
  DEPENDENT=B/ENTER X Y
SUBTITLE 'COMPUTE CONVENTIONAL CANONICAL ANALYSIS'
MANOVA X Y WITH A B/PRINT=DISCRIM(RAW,STAN,COR,ALPHA(1.0))
  SIGNIF(DIMENR EIGEN MULTIV)/DESIGN/
SUBTITLE 'PLOT SYNTHETIC VARIABLE SCORES FOR FUNCTION I'
SCATTERGRAM CRITCOMP (-6,3) WITH PREDCOMP (-3,6)
STATISTICS ALL
OPTIONS 4
SUBTITLE '**SHOW IMPORT OF STRUCTURE COEFS IN THE REG CASE'
REGRESSION VARIABLES=CRITCOMP PREDCOMP A B/DESCRIPTIVES=DEFAULTS/
  CRITERIA=TOLERANCE(.0001)/DEPENDENT=CRITCOMP/ENTER A B
SUBTITLE '##FIND BETA TO RESIDUALIZE ZA USING ZB'
REGRESSION VARIABLES=CRITCOMP PREDCOMP A B/DESCRIPTIVES=DEFAULTS/
  CRITERIA=TOLERANCE(.0001)/DEPENDENT=A/ENTER B
SUBTITLE '$$ILLUSTRATE THAT FUNCTION COEFS REFLECT PARTIAL CORR'
REGRESSION VARIABLES=CRITCOMP PREDCOMP A B ZARESI/
  CRITERIA=TOLERANCE(.0001)/DEPENDENT=CRITCOMP/ENTER ZARESI
```

Table 3
Selected Canonical Analysis Results

	Function I		Function II		
	Stn Fun	Struct	Stn Fun	Struct	Communality
ZX	-1.44986	-.69607	-.01281	.71798	1.000008
ZY	1.04101	-.00884	1.00924	.99996	.999998
ZA	-1.58021	-.61831	.02918	.78593	.999993
ZB	1.24215	.08146	.97723	.99983	1.006295
Rc	.99999		.02557		

Table 4
"Synthetic" Variate Scores for Function I

	ZX	ZY	ZA	ZB	CRITCOMP	PREDCOMP	CRITxPRED
	-1.35287	.63850	.00000	1.04326	1.29678	1.29589	1.680484
	-.44490	-1.26448	.00000	-.53744	-.67129	-.66758	.448139
	.00908	-.01252	-.81650	-1.06434	-.02620	-.03183	.000833
	.46306	1.23944	-.81650	-.53744	.61889	.62266	.385358
	1.32563	.67606	1.63299	1.09595	-1.21819	-1.21913	1.485131
Sum							3.999947

Note. The sum of the cross-products (3.999947) divided by n-1 (4) is, within rounding error, the canonical correlation.

Table 5
Calculation of Structure and Index Coefficients

	ZX	CRITCOMP	PREDCOMP	XSTRUC	XINDEX
	-1.35287	1.29678	1.29589	-1.75438	-1.75317
	-.44490	-.67129	-.66758	.29866	.29701
	.00908	-.02620	-.03183	-.00024	-.00029
	.46306	.61889	.62266	.28658	.28833
	1.32563	-1.21819	-1.21913	-1.61487	-1.61612
Sum				-2.78425	-2.78424

Note. The sum of the cross-products of "ZX" and "CRITCOMP" (-2.78425) divided by n-1 (4) is (-.69606), within rounding error, the structure coefficient of "ZX" on Function I. The sum of the cross-products of "ZX" and "PREDCOMP" (-2.78424) divided by n-1 (4) is (-.69606), within rounding error, the index coefficient of "ZX" on Function I.

Table 6
Hypothetical Data from Thompson (1985b)

Y	X	A	B	APRIME	A1	A2	B1	A1B1	A2B1	BT1	BT2
1	11	5	1	2	1	-1	-1	-1	1	0	1
2	5	3	1	1	-1	-1	-1	1	1	1	0
3	2	2	1	1	-1	-1	-1	1	1	1	0
4	8	8	0	2	1	-1	1	1	-1	0	1
5	4	4	0	1	-1	-1	1	-1	-1	1	0
6	12	10	1	3	0	2	-1	0	-2	0	0
7	7	6	1	2	1	-1	-1	-1	1	0	1
8	1	1	0	1	-1	-1	1	-1	-1	1	0
9	9	12	0	3	0	2	1	0	2	0	0
10	3	7	0	2	1	-1	1	1	-1	0	1
11	6	9	0	3	0	2	1	0	2	0	0
12	10	11	1	3	0	2	-1	0	-2	0	0

Table 7
SAS File to Analyze Table 6 Data

```
TITLE '$$$ SHOW CANONICAL SUBSUMES ALL 1985 AERA ERIC #ED262073';
DATA MULTI;
  INFILE DEMO7257;
  INPUT Y 1-2 X 4-5 A 7-8 B 10 APRIME 12 A1 14-15 A2 17-18
  B1 20-21 A1B1 23-24 A2B1 26-27 BT1 30 BT2 32;
PROC PRINT; RUN;
TITLE '1. CCA SUBSUMES T-TESTS #####';
PROC CANCORR ALL; VAR Y; WITH B;
PROC TTEST; CLASS B; VAR Y; RUN;
TITLE '2. CCA SUBSUMES PEARSON R #####';
PROC CANCORR ALL; VAR Y; WITH A;
PROC CORR PEARSON; VAR Y A; RUN;
TITLE '3. CCA SUBSUMESS FACTORIAL ANOVA #####';
PROC CANCORR ALL; VAR Y; WITH A1 A2 B1 A1B1 A2B1;
PROC CANCORR ALL; VAR Y; WITH B1 A1B1 A2B1;
PROC CANCORR ALL; VAR Y; WITH A1 A2 A1B1 A2B1;
PROC CANCORR ALL; VAR Y; WITH A1 A2 B1;
PROC ANOVA; CLASS APRIME B; MODEL Y=APRIME B APRIME*B; RUN;
TITLE '4. CCA SUBSUMES MULTIPLE R #####';
PROC CANCORR ALL; VAR Y; WITH X A B;
PROC REG SIMPLE; MODEL Y=X A B; RUN;
TITLE '5. CCA SUBSUMES DISCRIMINANT #####';
PROC CANCORR ALL; VAR BT1 BT2; WITH Y X;
PROC CANDISC ALL; VAR Y X; CLASS APRIME; RUN;
TITLE '6. CCA SUBSUMES FACTORIAL MANOVA #####';
PROC CANCORR ALL; VAR Y X; WITH A1 A2 B1 A1B1 A2B1;
PROC CANCORR ALL; VAR Y X; WITH B1 A1B1 A2B1;
PROC CANCORR ALL; VAR Y X; WITH A1 A2 A1B1 A2B1;
PROC CANCORR ALL; VAR Y X; WITH A1 A2 B1;
PROC ANOVA; CLASS APRIME B; MODEL Y X=APRIME B APRIME*B;
  MANOVA H=_ALL_; RUN;
```

Table 8
CCA Subsumes t-tests [Y by B(0,1)]

Canonical Analysis		t-test analysis	
Squared Rc	.149184	Mean Group 0	7.83333333
Rc	.386244	SD	2.78687400
lambda	.85081585	Mean Group 1	5.16666667
		SD	4.07021703
F	1.7534	t	1.3242
df	1/10	df	10
p calc	.2149	p calc	.2149

Table 9
CCA Subsumes Pearson r [Y with A]

Canonical Analysis		Pearson r	
Squared Rc	.320847		
Rc	.566434	r	.56643
lambda	.67915301		
F	4.7242		
df	1/10		
p calc	.0548	p calc	.0548

Table 10
Factorial ANOVA [Y by APRIME(1,3),B(0,1)]

Source	SOS	df	MS	F	p calc
APRIME	56.00000000	2	[28.000]	2.75	0.1417
B	21.33333333	1	[21.333]	2.10	0.1976
APRIME*B	4.66666667	2	[2.333]	0.23	0.8016
ERROR	61.00000000	6	10.16666667		
TOTAL	143.00000000	11			

Table 11
Canonical Analyses Using Four Models

Model	Predictors of Y	lambda
1	A1,A2,B1,A1B1,A2B1	.42657343
2	B1,A1B1,A2B1	.81818182
3	A1,A2,A1B1,A2B1	.57575758
4	A1,A2,B1	.45920746

Table 12
Conversion to ANOVA lambda's

Effect	Models	Conversion	Result
APRIME	1/2	.42657343/.81818182	.52136752
B	1/3	.42657343/.57575758	.74089068
APRIME*B	1/4	.42657343/.45920746	.92893401

Table 13
Conversion of lambda's to ANOVA F's

Source	[(1 - lambda) /lambda]	*(df error/df effect)	=F calc
APRIME	[(1 - .52136752)/.52136752]	*(6 / 2)	=
	.91803277	* 3	=2.7541
B	[(1 - .74089068)/.74089068]	*(6 / 1)	=
	.34972677	* 6	=2.0984
APRIME*B	[(1 - .92893401)/.92893401]	*(6 / 2)	=
	.07650272	* 3	=0.2295

Table 14
CCA Subsumes Multiple Correlation [Y with X, A, B]

Canonical Analysis		Regression Analysis	
Squared Rc	.699201	Squared R	.6992
Rc	.836182		
lambda	.30079890		
F	6.1986	F	6.199
df	3/8	df	3/8
p calc	.0175	p calc	.0175

Table 15

Function Coefficient and Beta Weight Conversions

Predictor	Function Coefficient	*Rc (or R)	=	Beta Weight/Rc (or R)	=	Function Coefficient
X	-1.1869	*.836182	=	-.99[illegible]4/.836182	=	-1.1869
A	1.5463	*.836182	=	1.2930/.836182	=	1.5463
B	0.1458	*.836182	=	.1219/.836182	=	0.1458

Table 16

CCA Subsumes Discriminant [Y, X with BT1, BT2 or APRIME(1,3)]

Canonical Analysis		Discriminant Analysis	
Function I		Function I	
Squared Rc	.908466	Squared Rc	.908466
Rc	.953135	Rc	.953135
lambda	.08561996	lambda	.08561996
F	9.6701	F	9.6701
df	4/16	df	4/16
p calc	.0004	p calc	.0004
Function II		Function II	
Squared Rc	.064608	Squared Rc	.064608
Rc	.254181	Rc	.254181
lambda	.93539214	lambda	.93539214
F	0.6216	F	0.6216
df	1/9	df	1/96
p calc	.4507	p calc	.4507

Table 17

Conversion of Function Coefficients for Comparison

Canonical Correlation Analysis

	Func I				Result	Func II		Result
Y	0.6334	/	0.7827	=	0.8093	-0.7739	-0.7739	1.0000
X	0.7827	/	0.7827	=	1.0000	0.6226	-0.7739	-0.8045

Discriminant Function Analysis

	Func I				Result	Func II		Result
Y	1.8938	/	2.3401	=	0.8093	0.7238	0.7238	1.0000
X	2.3401	/	2.3401	=	1.0000	-0.5823	0.7238	-0.8045

<u>Note</u>. The conversion process is illustrated in Tatsuoka (1971, pp. 177-183).

Table 18

Factorial MANOVA [Y, X by APRIME(1,3),B(0,1)]

Source	lambda	F	df	p calc
APRIME	.03202016	11.47	4/10	.0009
B	.60902256	1.60	2/5	.2895
APRIME*B	.37811816	1.57	4/10	.2572

Table 19
Canonical Analyses Using Four Models

Model	Predictors of Y, X	lambda
1	A1,A2,B1,A1B1,A2B1	.02112986
2	B1,A1B1,A2B1	.65989239
3	A1,A2,A1B1,A2B1	.03469471
4	A1,A2,B1	.05588163

Table 20
Conversion to MANOVA lambda's

Effect	Models	Conversion	Result
APRIME	1/2	.02112986/.65989239	.03202016
B	1/3	.02112986/.03469471	.60902252
APRIME*B	1/4	.02112986/.05588163	.37811817

Table 21
Beta Weights and Structure Coefficients for
Regression Prediction of CRITCOMP with A and B

Predictor Variable	Beta	r with CRITCOMP		R		Structure Coefficient
A	-1.580199	-0.618	/	0.99999	=	-0.61800618
B	1.242139	0.018	/	0.99999	=	0.01800018

Table 22
Redundancy Calculations for Hypothetical Data

	Struc I	SQ	Struc II	SQ	Commun	Pooled Rd
	-0.69607	0.484513	0.71798	0.515495	1.000008	
	-0.00884	0.000078	0.99996	0.999920	0.999998	
SUM		0.484591		1.515415	2.000006	
Adequacy		0.242295		0.757707	1.000003	
Redundancy		0.242290		0.000492		0.242783
	-0.61831	0.382307	0.78593	0.617685	0.999993	
	0.08146	0.006635	0.99983	0.999660	1.006295	
SUM		0.388942		1.617345	2.006288	
Adequacy		0.194471		0.808672	1.003144	
Redundancy		0.194467		0.000525		0.194993
Rc SQ		0.99998		0.00065		

Table 23
Alternate Calculation of Pooled Coefficients

Criterion Variables	Predictor Variables	R	R SQ
X	A B	0.69630	0.48484
Y	A B	0.02705	0.00073
SUM			0.48557
Mean			0.24279
A	X Y	0.61864	0.38271
B	X Y	0.03153	0.00099
SUM			0.38370
Mean			0.19185

Table 24
Correlation Matrix Associated with Sexton et al. (in press)

Instrument/Variable	So	Ad	Mo	Co	Cg	Me
Battelle Developmental Inventory						
Social (So)						
Adaptive (Ad)	730					
Motor (Mo)	758	835				
Communication (Co)	731	831	821			
Cognitive (Cg)	652	846	845	850		
Bayley Scales of Infant Development						
Mental (Me)	742	851	896	879	934	
Psychomotor (Ps)	758	827	947	810	832	901

Note. Decimals omitted.

Table 25
Canonical Correlation Analysis Coefficients

Variable/ Coefficient	I Func	Stru	Sq Struct	II Func	Stru	Sq Struct	h^2
Social	0.09	0.79	62.54%	-0.07	0.10	1.08%	64%
Adaptive	0.02	0.89	78.96%	0.24	-0.03	0.09%	79%
Motor	0.49	0.97	93.77%	1.85	0.24	5.62%	100%
Communication	0.11	0.90	80.73%	-0.48	-0.18	3.39%	84%
Cognitive	0.36	0.94	88.71%	-1.61	-0.30	8.88%	98%
Adequacy			80.94%			3.85%	
Redundancy			76.23%			1.58%	
Rc^2			94.17%			41.07%	
Redundancy			89.36%			2.10%	
Adequacy			94.89%			5.11%	
Mental	0.60	0.98	96.67%	-2.22	-0.18	3.33%	100%
Psychomotor	0.42	0.96	93.11%	2.27	0.26	6.89%	100%

Table 26
Prediction of Criterion Composite Scores on Function I with Various Predictor Variable Combinations

Predictors Set	Variables	Rc^2
1.	So	0.58894
2.	Ad, Mo	0.90295
3.	Co, Cg	0.86800
4.	So & Ad, Mo	0.90522
5.	So & Co, Cg	0.89457
6.	Ad, Mo & Co, Cg	0.93881
7.	ALL	0.94173

Table 27
Calculation of Variance Partitions

Partition	Result
Unique to So	
-Rc sq 6 +Rc sq 7	
-0.93881 0.94173	0.00292
Unique to Ad, Mo	
-Rc sq 5 +Rc sq 7	
-0.89457 0.94173	0.04716
Unique to Co, Cg	
-Rc sq 4 +Rc sq 7	
-0.90522 0.94173	0.03651
Common to So & Ad, Mo	
-Rc sq 3 +Rc sq 5 +Rc sq 6 -Rc sq 7	
-0.86800 0.89457 0.93881 -0.94173	0.02365
Common to So & Co, Cg	
-Rc sq 2 +Rc sq 4 +Rc sq 6 -Rc sq 7	
-0.90295 0.90522 0.93881 -0.94173	-0.00065
Common to Ad, Mo & Co, Cg	
-Rc sq 1 +Rc sq 4 +Rc sq 5 -Rc sq 7	
-0.58894 0.90522 0.89457 -0.94173	0.26912
Common to So & Ad, Mo & Co, Cg	
+Rc sq 1 +Rc sq 2 +Rc sq 3	
0.58894 0.90295 0.86800	
-Rc sq 4 -Rc sq 5 -Rc sq 6 +Rc sq 7	
-0.90522 -0.89457 -0.93881 0.94173	0.56302

Table 28
Conventional Presentation of Variance Partitions

Partition	Set #1	Set #2	Set #3
Unique to So	0.00292		
Unique to Ad, Mo		0.04716	
Unique to Co, Cg			0.03651
Common to So & Ad, Mo	0.02365	0.02365	
Common to So & Co, Cg	-0.00065		-0.00065
Common to Ad, Mo & Co, Cg		0.26912	0.26912
Common to So & Ad, Mo & Co, Cg	0.56302	0.56302	0.56302
Sum of Partitions	0.58894	0.90295	0.86800

Table 29
SPSS-X File to Compute Canonical Commonality Analysis

```
TITLE 'COMMONALITY ANALYSIS FOR THOMPSON-MILLER--ERIC ED263151'
DATA LIST RECORDS=2/V1 TO V15 (7F7.5/8F7.5)
BEGIN DATA
 .13739 .12727 .13157 .08959 .08600 .11204 .08290
 .08751 .07668 .07807 .02122 .08142 .06569 .00673 .01820
END DATA
LIST VARIABLES=ALL/CASES=1
COMPUTE UX1=V1-V5
COMPUTE UX2=V1-V4
COMPUTE UX3=V1-V3
COMPUTE UX4=V1-V2
COMPUTE CX1X2=V4+V5-V11-V1
COMPUTE CX1X3=V3+V5-V10-V1
COMPUTE CX1X4=V2+V5-V9-V1
COMPUTE CX2X3=V3+V4-V8-V1
COMPUTE CX2X4=V2+V4-V7-V1
COMPUTE CX3X4=V2+V3-V6-V1
COMPUTE CX1X2X3=V11+V10+V8+V1-V15-V5-V4-V3
COMPUTE CX1X2X4=V11+V10+V7+V1-V14-V5-V4-V2
COMPUTE CX1X3X4=V10+V9+V6+V1-V13-V5-V3-V2
COMPUTE CX2X3X4=V8+V7+V6+V1-V12-V4-V3-V2
COMPUTE C1234=V15+V14+V13+V12+V5+V4+V3+V2
              -V11-V10-V9-V8-V7-V6-V1
COMPUTE AGE=UX1+CX1X2+CX1X3+CX1X4+CX1X2X3+CX1X2X4+CX1X3X4+C1234
COMPUTE LOCUS=UX2+CX1X2+CX2X3+CX2X4+CX1X2X3+CX1X2X4+CX2X3X4+C1234
COMPUTE NEWREL=UX3+CX1X3+CX2X3+CX3X4+CX1X2X3+CX1X3X4+CX2X3X4+C1234
COMPUTE CODES=UX4+CX1X4+CX2X4+CX3X4+CX1X2X4+CX1X3X4+CX2X3X4+C1234
COMPUTE MULTR=UX1+UX2+UX3+UX4+CX1X2+CX1X3+CX1X4+CX2X3+CX2X4+CX3X4+
              CX1X2X3+CX1X2X4+CX1X3X4+CX2X3X4+C1234
PRINT FORMATS  UX1 TO C1234,AGE TO MULTR(F7.5)
LIST VARIABLES=UX1 TO C1234/CASES=1
LIST VARIABLES=AGE TO MULTR/CASES=1
```

Table 30
Selected Canonical Results Associated with
Webber et al. (1987/1988) Report

Variable	I	II	III
MHLC			
Chance	.953	.244	-.178
Powerful Others	-.036	.676	.736
Internal	.300	-.695	.654
Rc	.536	.439	.343
CHLC			
Chance	.937	.231	-.263
Powerful Others	.024	.707	.707
Internal	.349	-.668	.656

Note. Since orthogonal factors scores were employed as the variables in both data sets, the structure and the functions coefficients for this analysis were identical, as explained in Thompson (1984, p. 23, 36).

Table 31

Simultaneous Orthogonal Rotation of Table 30 Results

Variable	I	II	III
MHLC			
Chance	.999	.026	.035
Powerful Others	-.035	.010	.999
Internal	-.026	.999	-.010
CHLC			
Chance	.999	-.026	-.035
Powerful Others	.035	-.010	.999
Internal	.026	.999	.010

Table 32

Inter Variate Score Correlation Coefficients

	I	II	III
I	.522	.044	.015
II	.044	.408	-.046
III	.015	-.046	.389

Note. Adjusted correlation coefficients are on the diagonal.

Figure 1
Scattergram of Canonical Composite Scores on Function I

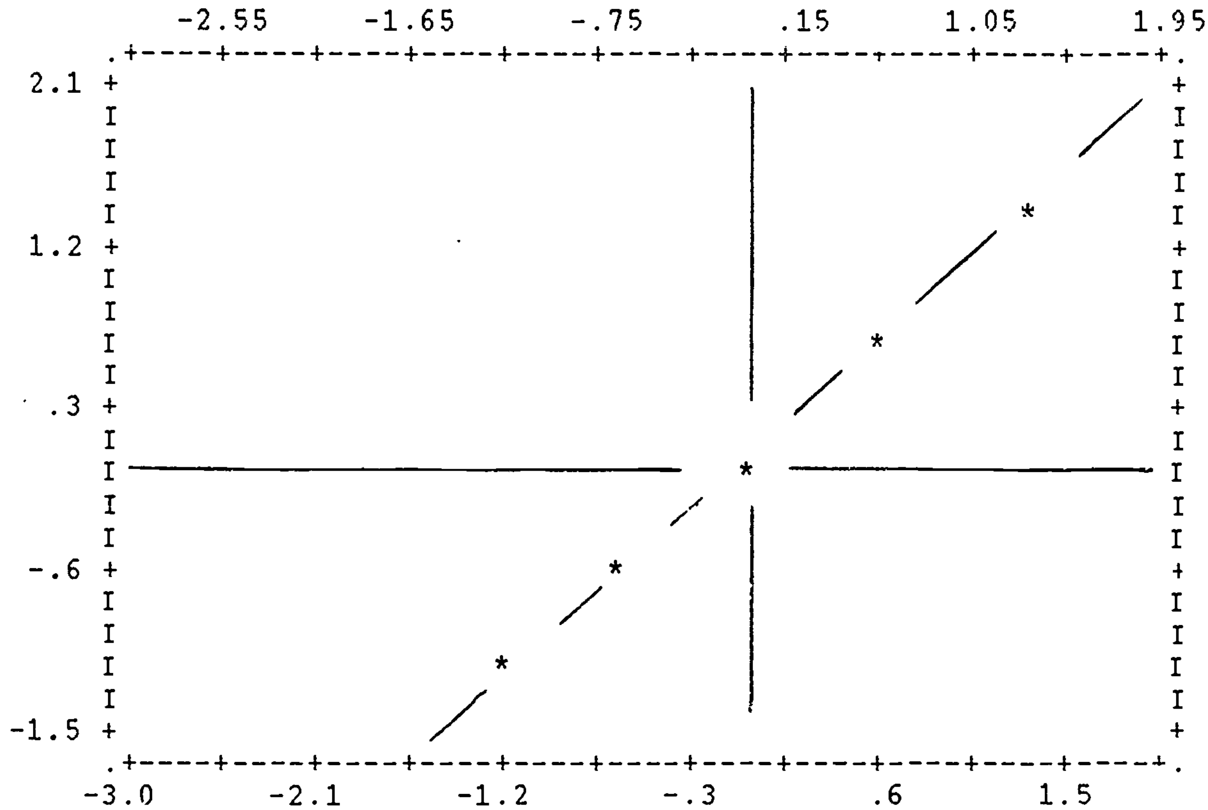
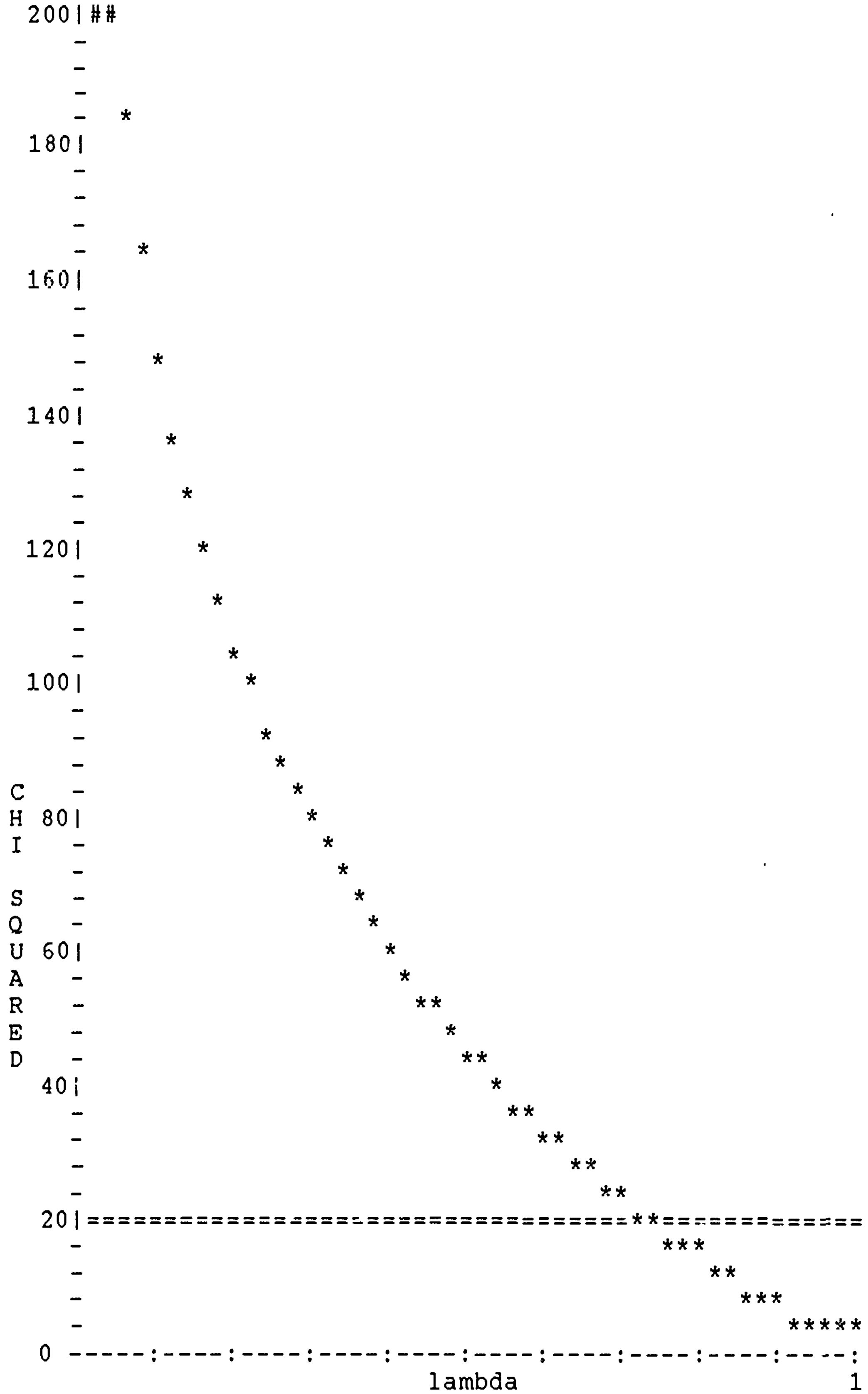


Figure 2
Partial Output from Program CANPOW: Actual or Expected n=70



Note. '=' = critical test statistic at declared alpha.